[8] Huber, P. W. and Kantrowitz, A., "Heat capacity lag measurements in various gases," J. Chem. Phys. **15**, 275–284 (1947).

[9] Hurle, I. R., private communication (April 16, 1964).

[10] Millikan, R. C. and White, D. R., "Vibrational relaxation in mixtures, including vibrational exchange," J. Chem. Phys. (to be published).

[11] See, for example, Callear, A. B., "Vibrational relaxation of nitric oxide," Discussions Faraday Soc. **33**, 28–36 (1962); also Rapp, D. and Englander-Golden, P., "Resonant and near-resonant vibrational-vibrational energy transfer between molecules in collisions," J. Chem. Phys. **40**, 573–575 (1964); note Erratum in J. Chem. Phys. **40**, 3120 (1964).

[12] White, D. R. and Millikan, R. C., "Oxygen vibrational relaxation in O_2-H_2 mixtures," J. Chem. Phys. **39**, 2107–2108 (1963).

[13] White, D. R., "Structure of gaseous detonation. III. Density in the induction zone of hydrogen detonation," Phys. Fluids **6**, 1011–1015 (1963).

[14] White, D. R. and Moore, G. E., "Structure of gaseous detonation. IV. Induction zone studies in H_2-O_2 and CO-O_2 mixtures," *Tenth Symposium (International) on Combustion* (Mono of Maryland, Baltimore, Md., to be published); also General Electric Research Lab. Rept. RL-64-3654C, Schenectady, N.Y. (May 1964).

Weak Interacting Hypersonic Flow over Cones

W. Hoelmer* and M. Saarlas†
University of Cincinnati, Cincinnati, Ohio

INTERACTING hypersonic laminar boundary-layer flow over cones has been of practical interest for a number of years. The principal methods of computing the induced pressures on the body have been given by Probstein[1] and Talbot et al.[2] The latter report also offers experimental data at moderate supersonic speeds verifying the theoretical method. Unfortunately, the foregoing methods are either too approximate or too cumbersome for practical calculations. Also, experimental data do not seem to exist for higher Mach numbers. It is the purpose of this note to simplify the computational procedure (to the point where only a simple interpolation is required) and to communicate some experimental data at $M = 14$ obtained in the Aerospace Research Laboratories (ARL) 20-in. hypersonic tunnel.

1. Simplification of the Computational Procedure

Several different methods have been proposed in the forementioned papers to evaluate the boundary-layer induced pressures on the cone surface for the weak interaction region, $\theta_c > \tan^{-1}(d\delta^*/dx)$, where θ_c is the cone semiangle and δ^* is the displacement thickness. A brief summary of these methods is pertinent before the present technique is given.

Probstein determined pressure as a Taylor series expansion in powers of $d\delta^*/dx$ and derived the coefficients from inviscid hypersonic similarity law of Lees.[3] The displacement thickness slope was obtained from flat-plate results of Lees and Probstein.[4] Although this method yields less accurate results than the second method of Talbot discussed below, it has the advantage of being relatively simple to use.

Talbot's first method (TC_∞ method) evaluates the local displacement thickness slope based on inviscid cone surface values instead of actual local boundary-layer edge values. The expression for displacement thickness comes from Crocco's flat-plate results.[5] The pressure is obtained directly then from Kopal tables for the effective cone angle $\theta_c + \tan^{-1}(d\delta^*/dx)$. This approximation will result in an overestimation of the induced pressure.[2]

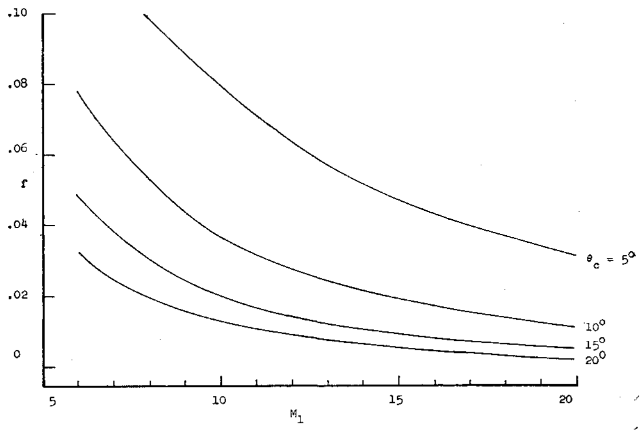

Fig. 1 f vs freestream Mach number M_1.

For more accurate calculations (TC method), the foregoing "inviscid" tangent cone method has been replaced by a semigraphical procedure to take local boundary-layer edge values into account, which gives a more precise estimate of the effective local cone angle. Although this procedure yields accurate results, it is cumbersome to apply to any particular problem.

In the present note, by using an inverse approach, the iteration method suggested by Talbot has been simplified to a one-step calculation. The details of the procedure are given in Ref. 6; here only a brief outline is included. In this technique, an equation is derived which demonstrates the functional relationship between displacement thickness slope, freestream Mach number, cone semivertex angle, wall- to adiabatic-wall temperature ratio, and hypersonic interaction parameter based on freestream conditions.

The local displacement thickness can be written as[2]

$$\frac{d\delta^*}{dx} = \frac{\bar{\chi}_2}{3^{1/2}M_2}\left[\frac{0.968}{M_2{}^2}\frac{T_w}{T_2} + 0.145\,(\gamma - 1)\right] \qquad (1)$$

where

$$\bar{\chi}_2 = (C_2)^{1/2}M_2{}^2/(R_{ex_2})^{1/2}$$

is the hypersonic interaction parameter based on conditions at the edge of the boundary-layer ($\delta^* \simeq \delta$). C_2 is the Chapman-Rubesin constant $(\mu/\mu_w)\,(T/T_w)$. If one introduces now the pressure, temperature, and velocity from inviscid hypersonic similarity results of Lees,[3] then the hypersonic interaction parameter, based on freestream conditions, can be written as follows (subscripts 1, 2, and c refer to freestream, edge of boundary layer, and cone surface, respectively):

$$\bar{\chi}_1 = \frac{M_2}{AF}\frac{d\delta^*}{d\chi} = \frac{M_2}{AF}\tan\theta_\delta \qquad (2)$$

where

$$A = \frac{0.968}{M_2{}^2}\frac{T_w}{T_{aw}}\left(1 + (Pr)^{1/2}\frac{\gamma - 1}{2}M_2{}^2\right) + 0.145\,(\gamma - 1)$$

$$F = F(M_1, \theta_2) = 3^{1/2}\left[\frac{M_1}{M_2}\right]^{5/2}\left[\frac{p_2}{p_1}\right]^{1/2}\left[\frac{T_1}{T_2}\right]^{3/4}$$

$$M_2 = M_2(M_1, \theta_2) = M_2(M_1, \theta_c + \theta_\delta) \text{ (tangent cone)}$$

Upon selection of M_1, θ_c, and after obtaining $\theta_2 = \theta_c + \theta_\delta$ from Eq. (2), the induced pressure increment

$$\frac{\Delta p}{p_c} = \frac{p_2 - p_c}{p_c} = \frac{p_2}{p_1}\frac{p_1}{p_c} - 1$$

Received June 12, 1964. This note is a part of the work supported under U. S. Air Force Contract 33(616) 8453 by the Aerospace Research Laboratories, Wright-Patterson Air Force Base.

* Research Assistant.

† Research Associate, Department of Aerospace Engineering. Member AIAA.

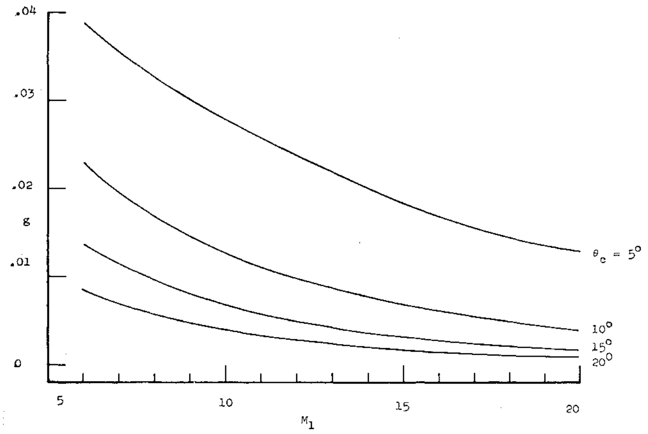

Fig. 2 g **vs freestream Mach number** M_1.

can now be computed from tables of Kopal, Sims,[6] or from Lees' similarity result.

The numerical problem was solved by an inverse procedure on IBM 7090 by selecting M_1, θ_c, T_w/T_{aw}, θ_δ, and by computing θ_2, $\bar{\chi}_1$ and $\Delta p/p_c$ from Eqs. (1–3). The results were found to be nearly linear with $\bar{\chi}_1$ (for constant M_1, θ_c, and T_w/T_{aw}) and can be expressed as

$$\Delta p/p_c = (fT_w/T_{aw} + g)\bar{\chi}_1$$

where

$$f = f(M_1, \theta_c) \qquad \text{Fig. 1}$$
$$g = g(M_1, \theta_c) \qquad \text{Fig. 2}$$

For $\theta_c > 10°$, linear interpolation will suffice, otherwise a crossplot (e.g., f vs θ_c at given M_1) is needed.

2. Experimental Results

A 10° semivertex angle sharp nosed cone (nose radius 0.001 in.) was tested at zero angle of attack at a nominal Mach number of 14 in the ARL 20-in. wind tunnel. The model was an uncooled tellurium copper cone with the wall thickness 0.125 in. to assure a nearly isothermal surface. The 7-in.-long model contained 7 pressure ports 0.0425 in. in diameter and 2 chromel-alumel thermocouples. Pressure was measured by Hastings-Raydist DV-13 heat conducting gages, and it was recorded on the microsadic magnetic tape, thus permitting individual before-after calibration checks. The tests were conducted at nominal stagnation pressures of 800, 1200, and 1600 psia with stagnation temperatures varying between 1800° and 1900°R. The method of data recording and tunnel conditions are available in detail in Refs. 5 and 7.

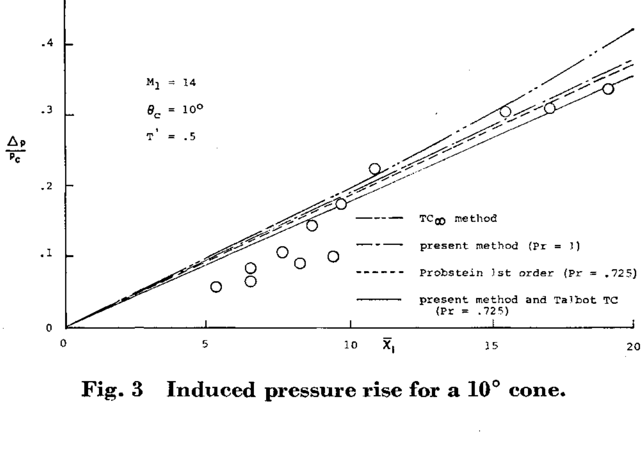

Fig. 3 Induced pressure rise for a 10° cone.

The results are indicated in Fig. 3. The greatest error in the prediction of the induced pressure increment (solid line) corresponds to an error in the surface pressure reading of at most 7%. Considering the magnitude of the static pressures that had to be measured (of order of 1 mm Hg) and the available instrumentation (sensitivity 0.1 mm Hg), it is concluded that all points fall well within the experimental scatter.

References

1 Probstein, R. F., "Interacting hypersonic laminar boundary layer flow over a cone," TR AF 2798/1, Brown Univ. (March 1955).

2 Talbot, L., Koga, T., and Sherman, P. M., "Hypersonic viscous flow over slender cones," NACA TN 4327 (1958).

3 Lees, L., "Note on the hypersonic similarity law for an unyawed cone," J. Aeronaut. Sci. 18, 700–702 (1951).

4 Lees, L. and Probstein, R. F., "Hypersonic viscous flow over a flat plate," Princeton Univ. Aeronautical Engineering Lab. Rept. 195 (April 1952).

5 Hoelmer, W. and Saarlas, M., "The weak interaction theory for hypersonic flow over cones," Aerospace Research Labs. Rept. ARL 64-37 (March 1964).

6 Sims, J. L., "Supersonic flow around right circular cones-tables for zero angle of attack," Army Ballistic Missile Agency Rept. DA-TR-11-60 (1960).

7 Brown, D. L., Hoelmer, W., Tepe, F. R., Jr., and Token, K. H., "Theoretical operating ranges and calibration results of the ARL twenty inch hypersonic wind tunnel," Aeronautical Research Labs. Rept. ARL 63-189 (October 1963).

Thermodynamic Properties for Imperfect Air and Nitrogen to 15,000°K

Clark H. Lewis* and C. A. Neel†
ARO, Inc., Arnold Air Force Station, Tenn.

The recently published thermodynamic properties (T, $\log\rho$, E/RT, H/RT, S/R, $\log p$, Z) for imperfect‡ air and nitrogen as given by Hilsenrath and Klein[1,2] have been used to compute the specific heat (C_p/R, C_v/R, and $\gamma = C_p/C_v$) and speed of sound [$\gamma_E \equiv (\partial \log p/\partial \log\rho)_S$, $a = (\gamma_E\, p/\rho)^{1/2}$] data by Lewis and Neel.[3,4] The data for air§ and nitrogen are given in the ranges $T = 1500(100)15{,}000°$K, $\log\rho = -7(0.2)\,2.2$ and $T = 2000(100)15{,}000°$K, $\log\rho = -7(0.2)2.4$. The data of Hilsenrath and Klein differ from the previously reported perfect-gas data of Hilsenrath, Klein, and Woolley[5] at densities where the effects of intermolecular forces are important. In general, imperfect gas effects are important only at densities above 1 amagat, i.e., the nondimensional density $\rho = \bar{\rho}/\bar{\rho}_a$ where $\bar{\rho}_a$ is the density at 1 atm and $T = 273.15°$K.

The specific heat and speed of sound data for nitrogen in the ranges $T = 100(100)2200°$K and $\log\rho = -7(0.2)2.4$ have

---

Received June 10, 1964. This work was sponsored by the Arnold Engineering Development Center, Air Force Systems Command U. S. Air Force, under Contract No. AF 40(600)-1000 with ARO, Inc., Operating Contractor, Arnold Engineering Development Center.

* Engineer, Hypervelocity Branch, von Karman Gas Dynamics Facility. Member AIAA.

† Member, Scientific Computing Branch, Engineering Support Facility.

‡ A perfect gas will denote one obeying $p = Z\rho RT$ which includes dissociation and ionization neglecting intermolecular effects. An imperfect gas obeys $p = Z\rho RT$ but includes dissociation, ionization and intermolecular (unless otherwise noted only two-body interaction) forces.

§ A multicolored Mollier diagram for imperfect air has been prepared by members of the staff of the von Karman Gas Dynamics Facility and is available on written request.